

The role of elastic and inelastic processes in the temperature dependence of Hall induced resistance oscillations in strong magnetic fields.

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Abstract

We develop a model of magnetoresistance oscillations induced by the Hall field in order to study the temperature dependence observed in recent experiments. The model is based on the solution of the von Neumann equation incorporating the exact dynamics of two-dimensional damped electrons in the presence of arbitrarily strong magnetic and dc electric fields, while the effects of randomly distributed neutral and charged impurities are perturbatively added. Both the effects of elastic impurity scattering as well as those related to inelastic processes play an important role. The theoretical predictions correctly reproduce the main experimental features provided that the inelastic scattering rate obeys a T^2 temperature dependence, consistent with electron-electron interaction effects.

1. Introduction

Magnetoresistance oscillations of two-dimensional electron systems (2DES), besides that Shubnikov-de Haas oscillations, have been a considerably active

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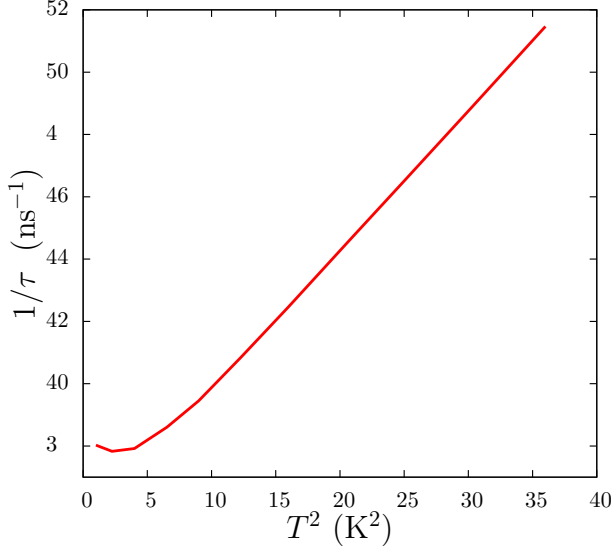


Figure 1: (color online). Single quantum scattering rate $1/\tau_q$ vs T^2

topic of research over the past decade. Microwave-induced resistance oscillations (MIRO) were registered [1, 2, 3, 4] in high mobility semiconductor 2DES samples subjected to microwave radiation and low magnetic fields. Similarly Hall field-induced resistance oscillations (HIRO) are observed in high mobility samples in the presence of a strong longitudinal dc-current excitation [5, 6, 7, 8]. Even though the mechanisms that produce MIRO and HIRO are different, both display periodic oscillations in $1/B$ that originates in the commensurability of the cyclotron frequency ω_c with a characteristic parameter of the system. Remarkably, in sufficiently clean samples the MIRO AND HIRO evolve into zero resistance or zero differential resistance states, respectively [9, 10]. Some models have been proposed to explain the main experimental features of HIRO. On one hand the *displacement model* relies on the impurity induced electron transitions between Landau levels (modified by the effects of the microwave radiation or the strong dc-excitation) [8, 11, 12] on the other, the *inelastic model* depends on the formation of a non-equilibrium distribution function induced by the Hall field [13].

Recently there has been considerable interest in the origin of the temperature dependence of MIRO [14, 15, 16] and HIRO [17]. In both cases the oscillations are best observed at $T \approx 1$ K and are smeared when T is of the order of a few Kelvin. These experiments show that the amplitude of the

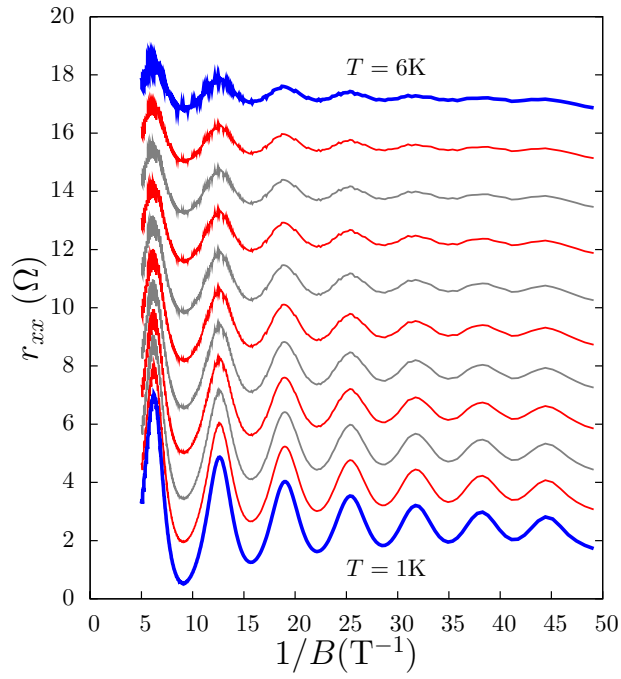


Figure 2: (color online). Longitudinal differential resistivity r_{xx} as a function of the inverse of the magnetic field $1/B$ for temperatures from $T = 1\text{K}$ (thick [blue] line) to $T = 6\text{K}$ (thick [blue] line) in steps of 0.5K (thin [gray and red] lines).

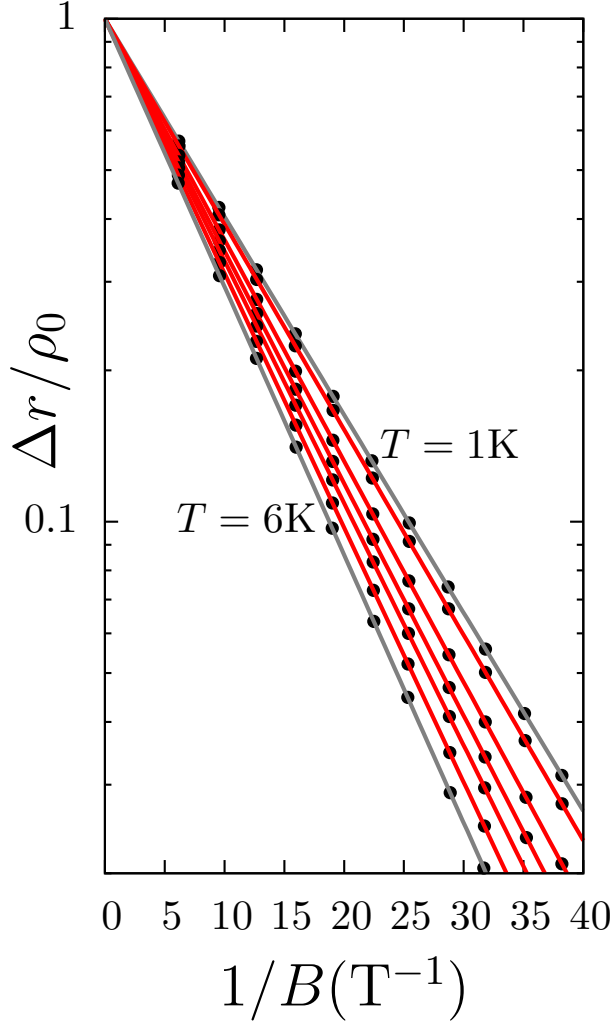


Figure 3: (color online). Normalized amplitude of the HIRO oscillations $\Delta r/\rho_0$ as a function of the inverse magnetic field $1/B$ for the temperatures $T = 1, 3, 4, 4.5, 5, 5.5, 6\text{K}$ for $\alpha_i = 25$. The points are obtained from the oscillation maxima of the plots in Fig. 2., whereas the lines are the predicted linear behavior of the Dingle factor, Eq.(22).

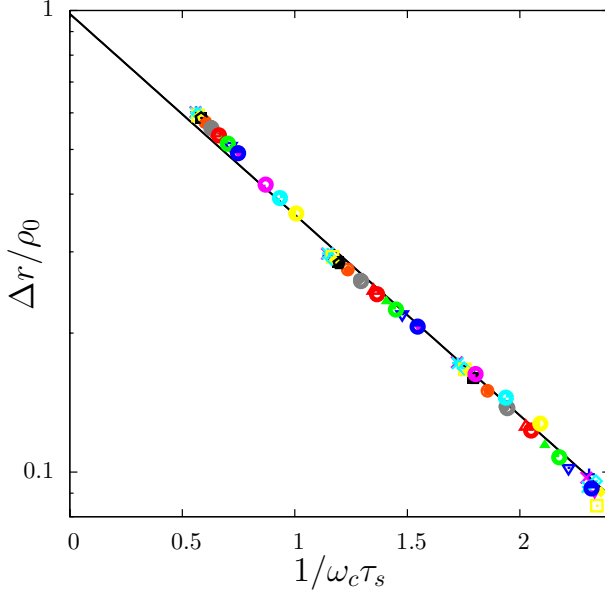


Figure 4: (color online). The normalized HIRO amplitude (normalized to the offset of the ρ_0) $\Delta r/\rho_0$ as a function of $1/\omega_c\tau_s$. The circles indicate $\alpha_i = 15$ and $\alpha_i = 25$, temperatures ranging from 1K to 6K in 0.5K intervals and magnetic fields between 0.02 and 0.2T. In this plot all data points obtained from our model for different temperatures, magnetic field and α values fall on the same universal line (solid line) prescribed by Eq. (21).

oscillations scale as $\exp(-aT^2/B)$. In this paper we present a model that correctly reproduces the temperature dependence of HIRO. Both the effects of elastic impurity scattering as well as those related to inelastic processes due to electron-electron scattering play an important role. The model is based on the exact solution of the von Neumann equation for a 2D damped electron gas subjected to arbitrarily strong magnetic and dc electric fields. In addition the model incorporates the weak effects of randomly distributed neutral and charged impurities through second order time dependent perturbation theory. This procedure yields a Kubo-like formula for the electric density current that incorporates the non-linear dependence on the electric field. Both inter- and intra-Landau level transitions contribute to the density current.

2. Model

Our starting point is the Hamiltonian for an electron in the effective mass approximation in two dimensions, subject to a uniform perpendicular magnetic field $\mathbf{B} = B\mathbf{k}$, an in-plane electric field $\mathbf{E} = E_x\mathbf{i} + E_y\mathbf{j}$, and the impurity scattering potential $V(\mathbf{r})$. Therefore the electron dynamics is governed by the total Hamiltonian $H = H_e + V$, with

$$H_e = H_0 + e\mathbf{E} \cdot \mathbf{x}, \quad (1)$$

here $H_0 = \mathbf{\Pi}^2/2m$, m is the effective electron mass, e is the electron's charge, $\mathbf{\Pi} = \mathbf{p} + e\mathbf{A}$ is the velocity operator and the vector potential for an homogeneous magnetic field in the symmetric gauge is given as $\mathbf{A} = (-By, Bx)/2$. We consider the impurity scattering potential to be composed of a combination of short-range uncorrelated neutral delta scatterers and charged scatterers

$$V(\mathbf{r}) = e^{-\Gamma|t|} \sum_i^{N_n} \int \frac{d^2q}{(2\pi)^2} V_n(q) \exp[i\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}_i)] \\ + e^{-\Gamma|t|} \sum_j^{N_c} \int \frac{d^2q}{(2\pi)^2} V_c(q) \exp[i\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}_j)], \quad (2)$$

where \mathbf{r}_i is the position of the i th neutral scatterer, \mathbf{r}_j is the position of the j th charged impurity and N_n and N_c are the number of neutral and charged impurities. The explicit form of the Fourier coefficients $V_n(q)$ and $V_c(q)$ depends on the nature of the scatterers. Neutral scatterers are characterized by a constant[18, 19, 12]

$$V_n(q) = \frac{2\pi\hbar^2\alpha}{m}, \quad (3)$$

where α is a dimensionless parameter. The explicit calculations only allows us to calculate the product $\alpha^2 N_n$, hence, without loss of generality, it is set as $\alpha \equiv 1$. The potential for charged impurities is modified by screening effects and takes the form

$$V_c(q) = \frac{\pi\hbar^2}{m} \frac{e^{-qd}}{1 + \frac{q}{q_{TF}}}, \quad (4)$$

where d is the thickness of the doped layer, ϵ_b is the relative permittivity of the surrounding media, and the Thomas-Fermi wave number is given by

$$q_{TF} = \frac{e^2 m}{2\pi\epsilon_0\epsilon_b\hbar^2}. \quad (5)$$

The adiabatic switching of the impurity potential at the initial time $t_0 \rightarrow -\infty$ is controlled by the factor $\exp(-\Gamma|t|)$.

Now we turn to the calculation of the density current. It is computed as the time and thermal average of the velocity operator

$$\mathbf{J} = \left[\frac{e}{TS} \int_{-T/2}^{T/2} dt \text{Tr} [\rho(t) \mathbf{\Pi}] \right]_{T \rightarrow \infty}, \quad (6)$$

where S is the sample surface, and $\rho(t)$ is the density matrix operator obtained from the solution of the von Neumann's equation $i\hbar\partial\rho/\partial t = [H, \rho]$. This equation is solved by means of a series of unitary transformations [23]. The magnetic and electric field part of the Hamiltonian are treated exactly by means of the first unitary transformations, that is expressed in terms of functions that solve the corresponding classical equations. Whereas the impurity potential is incorporated through second order perturbation theory. It is assumed that, in the absence of the impurity, potential the density matrix reduces to the equilibrium density matrix given by $\rho_0 = f(H_e)$, with $f(E)$ the Fermi distribution function.

The computation of Eq. (6) yields an explicit expression for the current density. It splits into the Drude and an impurity induced contribution:

$$\mathbf{J} = \mathbf{J}^D + \mathbf{J}^{imp}. \quad (7)$$

The Drude contribution is given by $\mathbf{J}^D = n_e e \mathbf{v}_d$, where n_e is the electron density and the drift velocity is given by

$$\mathbf{v}_d = \frac{e\tau_{tr}^{in}}{m} \frac{\mathbf{E} - \omega_c \tau_{tr}^{in} \mathbf{k} \times \mathbf{E}}{1 + (\omega_c \tau_{tr}^{in})^2}. \quad (8)$$

The transport inelastic scattering time τ_{tr}^{in} arises from a damping term $\mathbf{v}/\tau_{tr}^{in}$ that is added to the classical equation of motion in order to incorporate dissipative effects. The components of impurity induced density current can be expressed as

$$\mathbf{J}^{im} = \frac{e\omega_c}{\hbar^2} \sum_{\mu\mu'} \int \frac{d^2q}{2\pi} [f_{\mu,q/2} - f_{\mu',-q/2}] (n_n \mathbf{G}_{\mu\mu'}^n + n_c \mathbf{G}_{\mu\mu'}^c) \quad (9)$$

where $n_n = N_n/S$ and $n_c = N_c/S$ are the density of neutral and charged impurities; f is the Fermi distribution function evaluated at the tilted LL

energies $\mathcal{E}_{\mu,q} = \hbar\omega_c(\mu + 1/2) + \hbar\omega_q$ where $\omega_q = \mathbf{q} \cdot \mathbf{v}_d$. The function $\mathbf{G}_{\mu\mu'}$ is given by

$$\mathbf{G}_{\mu\mu'}^{n,c} = |V_{n,c}(q)|^2 |D_{\mu\mu'}(z_q)|^2 \frac{\mathbf{q}\Delta_{\mu\mu'} + 2\tilde{\mathbf{q}}\omega_c\Gamma}{\Delta_{\mu\mu'}^2 + 4\omega_c^2\Gamma^2}, \quad (10)$$

where $\mathbf{q} = (q_x, q_y)$ is the transfer momentum, and $\tilde{\mathbf{q}} = (q_y, -q_x)$ its dual, whereas

$$\Delta_{\mu\mu'} = [\omega_q + \omega_c(\mu - \mu')]^2 - \omega_c^2 + \Gamma^2. \quad (11)$$

Finally the matrix elements $D_{\mu,\nu}$ are given by

$$D_{\mu\mu'}(z_q) = \exp\left(-\frac{|z_q|^2}{2}\right) \begin{cases} z_q^{\mu-\mu'} \sqrt{\frac{\mu!}{\mu'}} L_{\mu'}^{\mu-\mu'}(|z_q|^2), \mu \geq \mu', \\ (-z_q^*)^{\mu'-\mu} \sqrt{\frac{\mu!}{\mu'}} L_{\mu'}^{\mu'-\mu}(|z_q|^2), \mu \leq \mu', \end{cases} \quad (12)$$

where $z_q = l_B(q_x - iq_y)/\sqrt{2}$, and $L_\nu^{\mu-\nu}$ denotes the associated Laguerre polynomial.

Though \mathbf{J}^D depends linearly on the electric field, \mathbf{J}^{imp} introduces a non-linear contribution through ω_q . From the Lorentzian form of the $G_{\mu\mu'}^{n,c}$ function in Eq. (10) it can be seen that the Γ parameter effectively controls the LL broadening[24]. We thus introduce disorder broadening effects through Γ , moreover, in order to take into account the LL width dependence on the magnetic field henceforth we shall consider[25, 26]

$$\Gamma = \sqrt{\frac{2\hbar^2\omega_c}{\pi\tau_q}}, \quad (13)$$

as expected we assume that the LL width depends on the quantum scattering times τ_q .

The differential conductivity tensor is calculated from $\sigma_{ij} = \partial J_i / \partial E_j$ and the differential resistivity tensor is obtained from the inverse of the conductivity: that is $r_{ij} = \sigma_{ij}^{-1}$.

HIRO experiments are carried out at a fixed longitudinal current density J_{dc} and zero transverse current. The longitudinal electric field E_x and the Hall field E_y are thus computed from the following implicit equations

$$J_x(E_x, E_y) = J_{dc}, \quad J_y(E_x, E_y) = 0, \quad (14)$$

where J_x and J_y are given by Eqs. (7-11). Given the complexity and the nonlinear character of the current density expressions, the solution of Eq.

(11) is computed by a recursive numerical iteration, until the accuracy goal $J_y/J_x \approx 1 \times 10^{-15}$ is reached [12].

In the present model the current density Eq. (7) depends directly on the transport inelastic scattering time Eq. (8), and the quantum scattering time Eq. (13). On the other hand, in order to make contact with experimental results we need to specify the mobility, that is given in terms of the total transport time $\mu = e\tau_{tr}/m$. Using the Matthiessen's rule it is possible to incorporate both inelastic and impurity contributions to the transport scattering time as

$$\frac{1}{\tau_{tr}} = \frac{1}{\tau_{tr}^{in}} + \frac{1}{\tau_{tr}^{imp}}. \quad (15)$$

The transport as well as the quantum scattering times associated to the impurity potentials are given by [18, 20]

$$\begin{aligned} \frac{1}{\tau_q^{imp}} = \frac{m}{\pi \hbar^3 k_F} & \left[n_n \int_0^{2k_F} dq \frac{|V_n(q)|^2}{\sqrt{1 - (\frac{q}{2k_F})^2}} \right. \\ & \left. + n_c \int_0^{2k_F} dq \frac{|V_c(q)|^2}{\sqrt{1 - (\frac{q}{2k_F})^2}} \right] \end{aligned} \quad (16)$$

and

$$\begin{aligned} \frac{1}{\tau_{tr}^{imp}} = \frac{m}{2\pi \hbar^3 k_F^3} & \left[n_n \int_0^{2k_F} q^2 dq \frac{|V_n(q)|^2}{\sqrt{1 - (\frac{q}{2k_F})^2}} \right. \\ & \left. + n_c \int_0^{2k_F} q^2 dq \frac{|V_c(q)|^2}{\sqrt{1 - (\frac{q}{2k_F})^2}} \right], \end{aligned} \quad (17)$$

where n_n and n_c are the neutral and charged impurity densities respectively, and k_F is the Fermi momentum. For the quantum inelastic time we assume that it arises from electron-electron scattering, that is known to be well estimated by the following expression [21, 22]

$$\frac{1}{\tau_q^{in}} = \frac{E_F}{\hbar} \left(\frac{k_B T}{E_F} \right)^2 \left[\ln \left(\frac{k_B T}{E_F} \right) + \ln \left(2 \frac{q_{TF}}{k_F} \right) + 1 \right]. \quad (18)$$

For typical experimental conditions, the following condition: $\ln\left(\frac{k_B T}{E_F}\right) + \ln\left(2\frac{q_{TF}}{k_F}\right) \ll 1$ holds; thus $1/\tau_q^{in}$ has mainly a T^2 behaviour

$$\frac{1}{\tau_q^{in}} \approx \frac{E_F}{\hbar} \left(\frac{k_B T}{E_F}\right)^2. \quad (19)$$

We have explicit expressions for the quantum impurity Eq. (16) and quantum inelastic Eq. (19) scattering times; hence it is straightforward to determine the total quantum scattering rate using

$$\frac{1}{\tau_q} = \frac{1}{\tau_q^{imp}} + \frac{1}{\tau_q^{in}}. \quad (20)$$

The determination of the transport inelastic time is more involved given the complex structure of the collision integral[22]. Thus, for simplicity, we shall assume a simple relation between the transport and quantum inelastic scattering times: $\tau_{tr}^{in} = \alpha_i \tau_q^{in}$ where α_i is a constant. Now we can use Eq.(15) to determine the total transport time.

3. Results and discussion

We consider a sample of electron surface density and mobility of $n_e = 3.7 \times 10^{11} \text{ cm}^{-2}$ and $\mu = 1 \times 10^7 \text{ cm}^2/\text{Vs}$ respectively, a doped layer of $d = 20 \text{ nm}$ with $n_c/n_n = 300$ and $n_n = 1.7 \times 10^7 \text{ cm}^{-2}$ that leads to a ratio between the quantum and transport impurity scattering times of $\tau_{tr}^{imp}/\tau_q^{imp} \approx 11$ according to Eqs. (16) and (17). The longitudinal current I is set to $80 \mu\text{A}$ and for the sample width we assume $w = 100 \mu\text{m}$; hence $J_{dc} = 0.8 \text{ A/m}$.

We have a formalism in which the nonlinear resistivity effects, as well as the relaxation rates can be consistently calculated, once the neutral and charged scattering potentials have been specified. The quantum scattering rate $1/\tau_q$ as a function of T^2 is presented in Fig. 1. As the temperature increases, we observe the linear behavior related to the inelastic scattering contribution Eq.(19). On the other hand, the low temperature limit is determined by the impurity scattering contribution in Eq.(16).

In Fig. 2 we present the longitudinal differential resistivity r_{xx} as a function of the inverse magnetic field for temperature values from 1K to 6K in 0.5K increments. We observe clear differential magnetoresistance oscillations periodic in $1/B$, we see up to the 6th cyclotron resonance. The HIRO originate from the inter-Landau level transitions induced by impurity scattering

and are governed by the ratio of the Hall to the cyclotron frequencies. We observe that the oscillation peaks appear near integer values of the dimensionless parameter $\epsilon = \omega_H/\omega_c$. Here $\hbar\omega_H = eE_H(2R_c)$ is the energy associated with the Hall voltage drop across the cyclotron diameter $R_c = v_F/\omega_c$ and $E_H = J_{dc}B/en_e$ is the Hall field. The oscillatory periodic pattern is clearly temperature independent, however we observe that the oscillation amplitude gradually decay with increasing temperature, and almost disappear at $T \approx 6K$. To further characterize the HIRO temperature dependance as well as its physical origin, we present in Fig. 3 the normalized HIRO amplitude $\Delta r/\rho_0 = (r_{xx} - r_0)/\rho_0$ corresponding to the same data shown in Fig. (1) and arrange it in a semilog plot as a function of the inverse of the magnetic field for temperatures ranging from $2K$ up to $6K$ in $1K$ steps in. Here ρ_0 is Drude resistivity and r_0 is the offset of the HIRO. It is interesting to observe that the oscillations amplitude clearly display an exponential decay. One also observe that the exponent grows with increasing temperature as T^2 and decreases with the magnetic field as $1/B$. The results of our detailed calculation reproduce the most important features of recent experiments and is also in good agreement with the approximated expression proposed in reference [13, 17]

$$r_{xx} = r_0 + \rho_0 \frac{(4\delta)^2}{\pi} \frac{\tau_t}{\tau_\pi} \cos\left(2\pi \frac{\omega_H}{\omega_c}\right) \quad (21)$$

where τ_π is the time describing electron backscattering off impurities and δ is the Dingle factor

$$\delta = \exp\left(-\frac{\pi}{\omega_c \tau_q}\right), \quad (22)$$

given in terms of the total quantum scattering time Eq.(20).

Our complete calculation depends on the total quantum scattering rate through the LL width Eq. (13) and also on the transport inelastic rate that appears on the term Drude $\mathbf{J}^D = n_e e \mathbf{v}_d$ with the drift velocity given by Eq. (8). Consequently the differential resistivity is expected to depend on the temperature, magnetic field and the parameter α_i that relates the transport and quantum inelastic scattering times: $\tau_{tr}^{in} = \alpha_i \tau_q^{in}$ where α_i . However Eqs. (21,22) suggest a simple relation for differential resistance on terms of the Dingle factor. In Fig. 4 we present the results of our model for the normalized HIRO amplitude $\Delta r/\rho_0$ normalized to the offset of the ρ_0 HIRO oscillations as a function of $1/\omega_c \tau_q$ where the different symbols (colors) indicate $\alpha_i = 15$ and $\alpha_i = 25$, temperatures ranging from $1K$ to $6K$ in $0.5K$

intervals and magnetic fields between 0.02 and 0.2T. In this plot all data points obtained for different temperatures, magnetic field and α_i values fall on the same universal line prescribed by Eqs. (21,22). Hence it is concluded that the HIRO temperature dependance originates from electron-electron contribution to the quantum scattering rate.

4. Conclusions

We have presented a model to explain the HIRO properties of high mobility 2DES, including its temperature dependence. The effects of the electric and magnetic part of the Hamiltonian were treated exactly, whereas the impurity potential is included by means of a second order perturbation calculation. Our model contains the effects of neutral delta scatterers and charged impurities. We introduced inelastic effects through the damping that appears in the solution of the classical equation of motion that leads to the Drude contribution Eq.(8). The T^2 dependence of τ_q^{in} is required in order to reproduce the correct temperature dependence of the HIRO amplitude, and is consistent with electron-electron inelastic processes.

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